Chapter 8 Solutions

Section 8.2

1. $x_1 = 45$, $n_1 = 69$, $\hat{p}_1 = x_1/n_1 = 0.652$, $x_2 = 57$, $n_2 = 73$, $\hat{p}_2 = x_2/n_2 = 0.781$, $\overline{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = 0.718$,

   $\overline{q} = 1 - \overline{p} = 0.282$, $\alpha = 0.05$.

   Here we test H_0: $p_1 = p_2$ against H_1: $p_1 < p_2$. This is a left tail test.

   $$\text{T.S.:}\quad z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\overline{p}\overline{q}}{n_1} + \dfrac{\overline{p}\overline{q}}{n_2}}} = \frac{(0.652 - 0.781) - 0}{\sqrt{\dfrac{0.718 \cdot 0.282}{69} + \dfrac{0.718 \cdot 0.282}{73}}} = -1.707\,.$$

   C.V.: $z = -1.645$

   p-value = 0.0436

   We reject the null hypothesis in support of the alternative. *The data support the claim that the proportion from population 1 is less than the proportion from population 2.*

3. $x_1 = 24$, $n_1 = 71$, $\hat{p}_1 = x_1/n_1 = 0.338$, $x_2 = 36$, $n_2 = 144$, $\hat{p}_2 = x_2/n_2 = 0.250$, $\overline{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = 0.279$,

   $\overline{q} = 1 - \overline{p} = 0.721$, $\alpha = 0.08$.

   Here we test H_0: $p_1 = p_2$ against H_1: $p_1 \neq p_2$. This is a two tail test.

   $$\text{T.S.:}\quad z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\overline{p}\overline{q}}{n_1} + \dfrac{\overline{p}\overline{q}}{n_2}}} = \frac{(0.338 - 0.250) - 0}{\sqrt{\dfrac{0.279 \cdot 0.721}{71} + \dfrac{0.279 \cdot 0.721}{144}}} = 1.353\,.$$

   C.V.: $z = \pm 1.75$

   p-value = 0.176

   We fail to reject the null hypothesis. *The there is not sufficient evidence to support the claim that the proportion from population 1 is not equal to the proportion from population 2.*

5. $x_1 = 575$, $n_1 = 1007$, $\hat{p}_1 = x_1/n_1 = 0.571$, $x_2 = 91$, $n_2 = 158$, $\hat{p}_2 = x_2/n_2 = 0.575$, $\overline{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = 0.572$,

   $\overline{q} = 1 - \overline{p} = 0.428$, $\alpha = 0.06$.

   Here we test H_0: $p_1 = p_2$ against H_1: $p_1 \neq p_2$. This is a two tail test.

   $$\text{T.S.:}\quad z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\overline{p}\overline{q}}{n_1} + \dfrac{\overline{p}\overline{q}}{n_2}}} = \frac{(0.571 - 0.575) - 0}{\sqrt{\dfrac{0.572 \cdot 0.428}{1007} + \dfrac{0.572 \cdot 0.428}{158}}} = -0.094\,.$$

   C.V.: $z = \pm 1.881$

   p-value = $2 \cdot (1 - 0.5359) = 2 \cdot 0.4641 = 0.9282$

   We fail to reject the null hypothesis. *The data do not give sufficient evidence to support rejection of the claim that the proportions of students successfully completing Math 71 and 71B are the same. The difference between the sample proportions was insignificant, as shown by the p-value of 0.925.*

7. $x_1 = 566$, $n_1 = 1086$, $\hat{p}_1 = x_1/n_1 = 0.521$, $x_2 = 67$, $n_2 = 112$, $\hat{p}_2 = x_2/n_2 = 0.598$, $\overline{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = 0.528$,

$\overline{q} = 1 - \overline{p} = 0.472$, $\alpha = 0.04$.

Here we test H_0: $p_1 = p_2$ against H_1: $p_1 < p_2$. This is a left tail test.

T.S.: $$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\overline{p}\overline{q}}{n_1} + \dfrac{\overline{p}\overline{q}}{n_2}}} = \frac{(0.521 - 0.598) - 0}{\sqrt{\dfrac{0.528 \cdot 0.472}{1086} + \dfrac{0.528 \cdot 0.472}{112}}} = -1.554\ .$$

C.V.: $z = -1.751$

p-value = 0.0606

We fail to reject the null hypothesis. *The data do not support the claim that the proportion of students successfully completing Math 71 is lower than the proportion of students who are successfully completing Math 71B. The difference between the two proportions was only marginally significant, with a p-value of 0.0606.*

Section 8.3

1. $\bar{x}_1 = 3.1$, $s_1 = 1.8$, $n_1 = 31 > 30$, $\bar{x}_2 = 2.4$, $s_2 = 0.9$, $n_2 = 37 > 30$, $\alpha = 0.05$.
   H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 < \mu_2$. This is a left tail test.
   T.S.: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(3.1 - 2.4) - (0)}{\sqrt{\dfrac{1.8^2}{31} + \dfrac{0.9^2}{37}}} = 1.969$,
   $V_1 = \dfrac{s_1^2}{n_1} = \dfrac{1.8^2}{31} = 0.1045$ and $V_2 = \dfrac{s_2^2}{n_2} = \dfrac{0.9^2}{37} = 0.0219$, $df = \dfrac{(V_1 + V_2)^2}{\dfrac{V_1^2}{n_1 - 1} + \dfrac{V_2^2}{n_2 - 1}} = 42.3$ which we round down to 42.
   C.V.: $t = -1.682$
   p-value = 0.9738 (Using 40 degrees of freedom).

   We fail to reject the null hypothesis, which means we cannot support the alternative, which was the original claim. This problem was strange, claiming that the mean for population 1 is smaller, when in fact, the sample mean from this population was larger. This is why the p-value was so large. *The data do not give sufficient evidence to support the claim that the mean from the first population is less than that of the second population. This is emphasized by the large p-value of 0.9738.*

3. $\bar{x}_1 = 0.096$, $s_1 = 0.0019$, $n_1 = 28$, $\bar{x}_2 = 0.095$, $s_2 = 0.0014$, $n_2 = 25$, $\alpha = 0.02$. Data appear to come from normal populations.
   H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$. This is a two tail test.
   T.S.: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(0.096 - 0.095) - (0)}{\sqrt{\dfrac{0.0019^2}{28} + \dfrac{0.0014^2}{25}}} = 2.196$,
   $V_1 = \dfrac{s_1^2}{n_1} = \dfrac{0.0019^2}{28} = 0.000000129$ and $V_2 = \dfrac{s_2^2}{n_2} = \dfrac{0.0014^2}{25} = 0.0000000784$,
   $df = \dfrac{(V_1 + V_2)^2}{\dfrac{V_1^2}{n_1 - 1} + \dfrac{V_2^2}{n_2 - 1}} = 49.3$ which we round down to 49.
   C.V.: $t = \pm 2.405$.
   p-value = $2 \cdot 0.0162 = 0.0324$ (Using 50 degrees of freedom).

   Once again, we fail to reject the null hypothesis, which is contrary to the original claim. *There is insufficient evidence to support the claim that the means of these two populations are different. Still, the difference between these sample means was significant, as indicated by the p-value of 0.0324.*

5. $\bar{x}_1 = 3547.67$, $s_1 = 3229.90$, $n_1 = 15$, $\bar{x}_2 = 5654.80$, $s_2 = 7530.65$, $n_2 = 15$, $\alpha = 0.01$. Data are assumed to come from normal populations.
H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$. This is a two tail test.
T.S.: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(3547.67 - 5654.8) - (0)}{\sqrt{\dfrac{3229.9^2}{15} + \dfrac{7530.65^2}{15}}} = -0.996$,
$V_1 = \dfrac{s_1^2}{n_1} = \dfrac{3229.9^2}{15} = 695483.6$ and $V_2 = \dfrac{s_2^2}{n_2} = \dfrac{7530.65^2}{15} = 3780713$,
$df = \dfrac{(V_1 + V_2)^2}{\dfrac{V_1^2}{n_1 - 1} + \dfrac{V_2^2}{n_2 - 1}} = 18.98$ which we round down to 18.
C.V.: $t = \pm 2.878$.
p-value = 2·(1 – 0.8347) = 2·0.1653 = 0.3306 (Here we rounded the T.S. to –1.0).

Once again, we fail to reject the null hypothesis, which is the claim we were asked to test. *There is insufficient evidence to warrant rejection the claim that the over-counted votes is equal to the mean under-counted votes. The difference between these sample means was insignificant, as indicated by the p-value of 0.3306.*

7. $\bar{x}_1 = 2.82$, $s_1 = 1.47$, $n_1 = 11$, $\bar{x}_2 = 0.90$, $s_2 = 0.74$, $n_2 = 10$, we have assumed the populations are normal, $\alpha = 0.05$.
H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 > \mu_2$. This is a right tail test.
T.S.: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(2.82 - 0.90) - (0)}{\sqrt{\dfrac{1.47^2}{11} + \dfrac{0.74^2}{10}}} = 3.831$
$V_1 = \dfrac{s_1^2}{n_1} = \dfrac{1.47^2}{11} = 0.1964$ and $V_2 = \dfrac{s_2^2}{n_2} = \dfrac{0.74^2}{10} = 0.0548$, $df = \dfrac{(V_1 + V_2)^2}{\dfrac{V_1^2}{n_1 - 1} + \dfrac{V_2^2}{n_2 - 1}} = 15.05$ which we

round down to 15.
C.V.: $t = 1.753$
p-value = 0.0009.

This time we clearly reject the null hypothesis in favor of the alternative, which was our original claim. *The sample data support the claim that the mean remission is longer for patients taking the experimental drug than for patients receiving the placebo. The difference between these averages was highly significant, as indicated by the small p-value of 0.0009.*