Chapter 4

Section 4.1

1. $\sum P(x)=1$, and $0 \le P(x) \le 1$ for all *x*, so the probabilities represent a probability distribution.

1. The totals needed are:

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
0	0.547	0	0
1	0.356	0.356	0.356
2	0.087	0.174	0.348
3	0.009	0.027	0.081
4	0.001	0.004	0.016
	1	0.561	0.801

The variance is $\sigma^2 = Var(x) = \sum x^2 \cdot P(x) - \mu^2 = 0.801 - 0.561^2 = 0.4863$,

The standard deviation is $\sigma = \sqrt{\sigma^2} = \sqrt{0.4863} = 0.6973$

3. The totals needed are:
(Note that the *x* values are winnings minus the bet amount.)

x	$P(x)$	$xP(x)$	$x^2P(x)$
-160	0.56	-89.6	14336
0	0.38	0	0
1440	0.06	86.4	124416
	1	-3.2	138752

The mean winning is $\mu = E(x) = \sum xP(x) = -3.20$. This means that there is an average loss of $3.20 overall.

5. The totals needed are:

x	$P(x)$	$xP(x)$
15000	0.6	9000
-5000	0.4	-2000
	1	7000

The expected return on the investment is $\mu = E(x) = \sum xP(x) = \$7,000$.

7. The first investment has a higher average return, so this is what she should pick.

9. The probability distribution is:

x	$P(x)$
0	0.125
1	0.375
2	0.375
3	0.125

11. The totals needed are:

x	$P(x)$	$xP(x)$	$x^2P(x)$
0	0.125	0	0
1	0.375	0.375	0.375
2	0.375	0.75	1.5
3	0.125	0.375	1.125
	1	1.5	3

The variance is $\sigma^2 = Var(x) = \sum x^2 \cdot P(x) - \mu^2 = 3 - 1.5^2 = 0.75$,

The standard deviation is $\sigma = \sqrt{\sigma^2} = \sqrt{0.75} = 0.8660$

13. $\mu_2 = 5$, $\sigma_2^2 = 6.5$.

16. This problem refers to the means and variances in *problems #14 and #15*.

Section 4.2

1. $_{10}C_4 = \dfrac{10!}{4!(10-4)!} = 210$

3. $\mu = np = 10 \cdot 0.30 = 3.0$, $\sigma = \sqrt{npq} = \sqrt{10 \cdot 0.30 \cdot 0.70} = 1.449$

5. $_{20}C_{13} = \frac{20!}{13!(20-13)!} = 77{,}520$

7. $\mu = np = 20 \cdot 0.45 = 9.0$, $\sigma = \sqrt{npq} = \sqrt{20 \cdot 0.45 \cdot 0.55} = 2.225$

9. n = 10, x = 1, success is S = "speaks English", $p = P(S) = 0.05$.

$P(x=1) = {}_nC_x p^x q^{n-x} = {}_{10}C_1 \cdot 0.05^1 \cdot 0.95^{10-1}$

$= \frac{10!}{1!(10-1)!} \cdot 0.05^1 \cdot 0.95^9 = 10 \cdot 0.05^1 \cdot 0.95^9 = 0.3151.$

11. $\mu = np = 27 \cdot 0.88 = 23.76$, $\sigma = \sqrt{npq} = \sqrt{27 \cdot 0.88 \cdot 0.12} = 1.6885$

13. n = 12, x = 8, success is S = "approval", $p = P(S) = 0.60$.

$P(x=8) = {}_nC_x p^x q^{n-x} = {}_{12}C_8 \cdot 0.60^8 \cdot 0.40^{12-8}$

$= \frac{12!}{8!(12-8)!} \cdot 0.60^8 \cdot 0.40^4 = 495 \cdot 0.60^8 \cdot 0.40^4 = 0.2128.$

15. Using the rule of complements, P(*at least one*) = 1 – P(*none*) = 1 – $P(x = 0)$

n = 5, x = 0, success is S = "rain", $p = P(S) = 0.20$.

$P(x=0) = {}_nC_x p^x q^{n-x} = {}_5C_0 \cdot 0.20^0 \cdot 0.80^{5-0}$

$= \frac{5!}{0!(5-0)!} \cdot 0.20^0 \cdot 0.80^5 = 1 \cdot 0.20^0 \cdot 0.80^5 = 0.3277.$

Returning to the original question,

P(*at least one*) = 1 – P(*none*) = 1 – $P(x = 0)$ = 1 – 0.3277 = 0.6723.